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Unifiable Supersymmetric Left-Right Model with E_6 Particle Content

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Abstract

A new supersymmetric gauge model is proposed with particle content chosen only from the **27** and **27*** representations of E_6. The gauge symmetry $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ is realized at the TeV energy scale and the gauge couplings converge to a single value at around 10^{16} GeV. A discrete $Z_4 \times Z_2$ symmetry leads to a generalized definition of lepton number and ensures the absence of tree-level flavor-changing neutral-current interactions at the electroweak energy scale.

Several years ago, in connection with superstring theory, a supersymmetric left-right model was proposed[1] which has the interesting property that the $SU(2)_R$ charged gauge boson W_R has a nonzero lepton number, together with new exotic quarks h of charge $-1/3$. Its many unconventional implications have been studied in a number of subsequent publications.[2-16] This model is potentially of great phenomenological interest, but only if the $SU(2)_R$ breaking scale M_R is low enough, say of order a few TeV. However, that poses a problem for the unification of gauge couplings. With the particle content of the original model[1] and the experimentally determined values of the gauge couplings at the electroweak energy scale, it is simply not possible for them to converge to a single value unless M_R is very high. This is a well-known general result for left-right models.[17] Hence new particles are necessary if unification of the gauge symmetry is to be achieved.[18]

To be compatible with its possible superstring antecedent, the particle content of the proposed supersymmetric model in this paper is assumed to consist of only components from the **27** and **27*** representations of E_6. [The **27** of E_6 decomposes into **16** + **10** + **1** of SO(10).] At the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ level, it is proposed that there are three copies of

$$Q = (u, d)_L \sim (3, 2, 1, 1/6), \qquad d_L^c \sim (\overline{3}, 1, 1, 1/3) \tag{1}$$

$$Q^c = (h^c, u^c)_L \sim (\overline{3}, 1, 2, -1/6), \qquad h_L \sim (3, 1, 1, -1/3), \tag{2}$$

one bidoublet

$$\eta = \begin{pmatrix} \eta_1^0 & \eta_2^+ \\ \eta_1^- & \eta_2^0 \end{pmatrix} \sim (1, 2, 2, 0), \tag{3}$$

six copies of

$$\Phi_L = (\phi_L^0, \phi_L^-) \sim (1, 2, 1, -1/2), \tag{4}$$

$$\Phi_R = (\phi_R^+, \phi_R^0) \sim (1, 1, 2, 1/2), \tag{5}$$

three copies of

$$\Phi_L^c \sim (1,2,1,1/2), \qquad \Phi_R^c \sim (1,1,2,-1/2), \tag{6}$$

and six singlets

$$N \sim (1,1,1,0). \tag{7}$$

Under SO(10), Q, Q^c, Φ_L, and Φ_R belong to the **16**, d^c, h, and η belong to the **10**, Φ_L^c and Φ_R^c belong to the **16***. Anomaly cancellation at the $\mathrm{SU(3)} \times \mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{U(1)}$ level is assured because of the anomaly-free combinations $Q + Q^c + \Phi_L + \Phi_R$, $d^c + h$, $\Phi_L + \Phi_L^c$, and $\Phi_R + \Phi_R^c$. The left-right gauge symmetry is broken spontaneously at $\mathrm{M_R}$ by the nonzero vacuum expectation values of Φ_R and Φ_R^c. The scale of soft supersymmetry breaking is assumed to coincide with $\mathrm{M_R}$. The effective particle content at the electroweak energy scale is assumed to be that of the (nonsupersymmetric) standard $\mathrm{SU(3)} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ model with two Higgs doublets.

Consider now the evolution of the gauge couplings to two-loop order. Generically,

$$\mu \frac{\partial \alpha_i(\mu)}{\partial \mu} = \frac{1}{2\pi} \left(b_i + \frac{b_{ij}}{4\pi} \alpha_j(\mu) \right) \alpha_i^2(\mu), \tag{8}$$

where $\alpha_i \equiv g_i^2/4\pi$ and b_i, b_{ij} are constants determined by the particle content contributing to α_i. The initial conditions are set at $M_Z = 91.187 \pm 0.007$ GeV[19] by the experimental values $\alpha^{-1} = 127.9 \pm 0.1$,[20] $\sin^2\theta_W = 0.2321 \pm 0.0006$,[21] and $\alpha_S = 0.120 \pm 0.006 \pm 0.002$.[22] Hence

$$\alpha_S^{-1}(M_Z) = 8.33 \begin{array}{c} +0.60 \\ -0.52 \end{array}, \quad \alpha_L^{-1}(M_Z) = 29.69 \pm 0.10, \quad \alpha_Y^{-1}(M_Z) = 98.21 \pm 0.15. \tag{9}$$

At $\mathrm{M_R}$, the matching conditions of the gauge couplings are

$$\alpha_L^{-1} = \alpha_R^{-1}, \quad \alpha_Y^{-1} = \alpha_R^{-1} + \alpha_X^{-1}, \tag{10}$$

where α_X refers to the U(1) gauge coupling of the left-right symmetry. Above $\mathrm{M_R}$, α_L and α_R will evolve together identically.

In the one-loop approximation, below $\mathrm{M_R}$,

$$b_S = -11 + \frac{4}{3}(3) = -7, \tag{11}$$

$$b_L = -\frac{22}{3} + \frac{4}{3}(3) + \frac{1}{6}(2) = -3, \tag{12}$$

$$b_Y = \frac{20}{9}(3) + \frac{1}{6}(2) = 7, \tag{13}$$

whereas above $\mathrm{M_R}$,

$$b_S = -9 + 2(3) + n_h = 0, \tag{14}$$

$$b_{LR} = -6 + 2(3) + n_{22} + n_\phi = 4, \tag{15}$$

$$\frac{3}{2}b_X = 2(3) + 3n_\phi + n_h = 18, \tag{16}$$

where $n_{22} = 1$, $n_h = 3$, and $n_\phi = 3$ refer respectively to the one bidoublet η, the three copies of $h + d^c$, and the three copies of $\Phi_L + \Phi_L^c + \Phi_R + \Phi_R^c$, and the factor $3/2$ for b_X comes from the normalization of the $\mathrm{U}(1)_X$ coupling within SO(10). Assuming that $\alpha_S^{-1} = \alpha_{LR}^{-1} = (3/2)\alpha_X^{-1}$ at the unification scale $\mathrm{M_U}$ and neglecting the two-loop coefficients b_{ij}, Eq. (8) can be solved for $\mathrm{M_R}$ and $\mathrm{M_U}$, *i.e.*

$$\ln \frac{\mathrm{M_R}}{M_Z} = \frac{\pi}{4}\left[3\alpha^{-1}(M_Z)\{1 - 5\sin^2\theta_W(M_Z)\} + 7\alpha_S^{-1}(M_Z)\right] < 1.66, \tag{17}$$

and

$$\ln \frac{\mathrm{M_U}}{M_Z} = \frac{\pi}{2}\left[\alpha^{-1}(M_Z)\sin^2\theta_W(M_Z) - \alpha_S^{-1}(M_Z)\right] > 32.45. \tag{18}$$

Hence $\mathrm{M_R} < 480$ GeV and $\mathrm{M_U} > 1.1 \times 10^{16}$ GeV. The upper bound of $\mathrm{M_U}$ is 1.9×10^{16} GeV, corresponding to $\mathrm{M_R} = \mathrm{M_Z}$. Note that these results are identical to those of a recently proposed extension of the conventional supersymmetric left-right model[18] because each corresponding b_i above $\mathrm{M_R}$ differs by the same amount, *i.e.* one.

The allowed parameter space opens up more in two loops. Using[23]

$$b_{ij} = \begin{pmatrix} -26 & \frac{9}{2} & \frac{11}{10} \\ 12 & 8 & \frac{6}{5} \\ \frac{44}{5} & \frac{18}{5} & \frac{104}{25} \end{pmatrix} \tag{19}$$

for α_S^{-1}, α_L^{-1}, and $(3/5)\alpha_Y^{-1}$ below $\mathrm{M_R}$, and

$$b_{ij} = \begin{pmatrix} 48 & 9 & 3 \\ 24 & 49 & \frac{15}{2} \\ 24 & \frac{45}{2} & \frac{45}{2} \end{pmatrix} \tag{20}$$

for α_S^{-1}, α_{LR}^{-1}, and $(3/2)\alpha_X^{-1}$ above $\mathrm{M_R}$, and solving Eq. (8) numerically with the proper boundary conditions at $\mathrm{M_U}$:

$$\alpha_U^{-1} - \frac{2}{3\pi} = \alpha_S^{-1} - \frac{1}{4\pi} = \alpha_{LR}^{-1} - \frac{1}{6\pi} = \frac{3}{2}\alpha_X^{-1}, \tag{21}$$

it is found that

$$1.0 \times 10^{16}\ \mathrm{GeV} < \mathrm{M_U} < 2.3 \times 10^{16}\ \mathrm{GeV} \tag{22}$$

with

$$1.3\ \mathrm{TeV} > \mathrm{M_R} > M_Z. \tag{23}$$

As an example, Fig. 1 shows the case with $\mathrm{M_R} = 1$ TeV and $\mathrm{M_U} = 1.1 \times 10^{16}$ GeV for the values $\alpha^{-1}(M_Z) = 127.9$, $\sin^2\theta_W(M_Z) = 0.2316$, and $\alpha_S(M_Z) = 0.112$.

In the original model,[1] there are three copies of the **27** representations of $\mathrm{E_6}$, *i.e.* three copies of the **16**, **10**, and **1** representations of SO(10). To arrive at the present model, two bidoublets (belonging to the **10**) are removed, but three copies of $\Phi_L + \Phi_R + \Phi_L^c + \Phi_R^c$ (belonging to the **16** + **16***) and three more singlets are added. This assumed modification at low energies is what makes this model unifiable despite having $\mathrm{M_R}$ at about 1 TeV. At the unification scale $\mathrm{M_U}$, there are presumably at least six copies of **27** and three copies of **27***. The missing components are assumed to be superheavy with masses of order $\mathrm{M_U}$.

The interactions of this model at the $\mathrm{SU(3) \times SU(2)_L \times SU(2)_R \times U(1)}$ level are assumed to obey a discrete $\mathrm{Z_4 \times Z_2}$ symmetry, under which the various superfields transform as given in Table 1. Consider first only those terms in the superpotential involving the quarks. Only three are allowed:

$$QQ^c\eta \quad = \quad dh^c\eta_1^0 - uh^c\eta_1^- + uu^c\eta_2^0 - du^c\eta_2^+, \tag{24}$$

$$
\begin{aligned}
Qd^c\Phi_{L(1)} &= dd^c\phi^0_{L(1)} - ud^c\phi^-_{L(1)}, && (25)\\
hQ^c\Phi_{R(1)} &= hh^c\phi^0_{R(1)} - hu^c\phi^+_{R(1)}. && (26)
\end{aligned}
$$

This means that the exotic heavy quark h gets its mass from the vacuum expectation value $\langle\phi^0_{R(1)}\rangle$, the ordinary quarks u and d get their masses from $\langle\eta^0_2\rangle$ and $\langle\phi^0_{L(1)}\rangle$ respectively, and the fermionic components of the $\mathrm{SU(2)_L}$ doublet (η^0_1, η^-_1) and singlet $\phi^+_{R(1)}$ can be identified as ordinary leptons,[1] such as (ν_τ, τ^-) and τ^+. Consequently, h picks up a nonzero lepton number $L = 1$ and since W^-_R converts h^c to u^c, it also has $L = 1$. This prevents $d - h$ as well as $W_L - W_R$ mixing, and since only one scalar vacuum expectation value contributes to the mass of each quark species, the absence of tree-level flavor-changing neutral-current interactions is also assured.

Consider next the terms involving Φ^c_L and Φ^c_R. There are three quadratic terms:

$$
\Phi_{L(4,5,6)}\Phi^c_{L(1,2,3)}, \qquad \Phi_{R(4)}\Phi^c_{R(1)}, \qquad \Phi_{R(5,6)}\Phi^c_{R(2,3)}. \tag{27}
$$

This means that $\Phi_{L(4,5,6)} + \Phi^c_{L(1,2,3)} + \Phi_{R(4,5,6)} + \Phi^c_{R(1,2,3)}$ can be assumed heavy with masses of order $\mathrm{M_R}$. There are also three cubic terms:

$$
\Phi_{L(2,3)}\Phi^c_{L(1,2,3)}N_L, \qquad \Phi_{R(1)}\Phi^c_{R(1)}N_R, \qquad \Phi_{R(2,3)}\Phi^c_{R(2,3)}N_R. \tag{28}
$$

This means that N_R should have $L = 0$ whereas $\Phi_{L(2,3)}$ can be assigned $L = 1$, $\phi^+_{R(2,3)}$ and N_L assigned $L = -1$. However, the quadratic terms N_LN_L and N_RN_R are also allowed. Hence additive lepton number is explicitly violated by the N_L Majorana mass terms, but they are the sole source of this violation, as is often the case when the standard model is extended to include neutral singlet leptons.

The remaining allowed terms in the superpotential all involve the bidoublet:

$$
\begin{aligned}
\eta\eta N_L &= \eta^0_1\eta^0_2 N_L - \eta^-_1\eta^+_2 N_L, && (29)\\
\Phi_{L(1)}\Phi_{R(1)}\eta &= \phi^-_{L(1)}\phi^+_{R(1)}\eta^0_1 - \phi^0_{L(1)}\phi^+_{R(1)}\eta^-_1 + \phi^0_{L(1)}\phi^0_{R(1)}\eta^0_2 - \phi^-_{L(1)}\phi^0_{R(1)}\eta^+_2, && (30)
\end{aligned}
$$

and

$$\Phi_{L(4,5,6)}\Phi_{R(2,3)}\eta. \tag{31}$$

This means that the fermionic component of η_1^0 (*i.e.* ν_τ) gets a seesaw mass through its coupling to N_L via $\langle\eta_2^0\rangle$ and the N_L Majorana mass. Since the τ lepton is identified as the fermionic components of η_1^- and $\phi^+_{R(1)}$, it gets a mass via $\langle\phi^0_{L(1)}\rangle$. The particle content of this model under $\mathrm{SU(3)}\times\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ is given in Table 2 together with the baryon number B, lepton number L, and R parity of the fermions $R_f = (-1)^{1+3B+L}$. Strictly speaking, because N_L has a Majorana mass, lepton number is conserved only multiplicatively.

The electron and muon are identified in this model as the fermionic components of $\phi^-_{L(2,3)}$ and $\phi^+_{R(2,3)}$, whereas ν_e and ν_μ are equated with the fermionic components of $\phi^0_{L(2,3)}$. The latter are coupled to N_L via $\langle(\phi^c_L)^0\rangle$, hence they acquire seesaw masses in the same way as ν_τ and all three neutrinos may mix with one another. On the other hand, e and μ are massless at tree level. To see how they acquire radiative masses, note that the soft supersymmetry-breaking term

$$\Phi_{L(2,3)}\Phi_{R(2,3)}\tilde{\eta} = \phi^-_{L(2,3)}\phi^+_{R(2,3)}\overline{\eta^0_2} - \phi^0_{L(2,3)}\phi^+_{R(2,3)}\eta^-_2 + \phi^0_{L(2,3)}\phi^0_{R(2,3)}\overline{\eta^0_1} - \phi^-_{L(2,3)}\phi^0_{R(2,3)}\eta^+_1 \tag{32}$$

involving only the scalar fields is allowed by the discrete $\mathrm{Z}_4\times\mathrm{Z}_2$ symmetry. Since $\phi^-_{L(2,3)}$ and $\phi^+_{R(2,3)}$ are now identified as the scalar supersymmetric partners of e and μ, the radiative mechanism[24, 25] of gaugino exchange allows e and μ to become massive via $\langle\eta_2^0\rangle$. Note that there is mixing between $\phi^-_{L(2,3)}$ and η_1^+ via $\langle\phi^0_{R(2,3)}\rangle$, hence e and μ also mix with τ through radiative corrections. In addition, because of the $\Phi_{L(4,5,6)}\Phi_{R(2,3)}\eta$ term, flavor-changing leptonic processes are possible at the TeV energy scale. Phenomenological details will be given elsewhere.

In summary, the proposed supersymmetric left-right model of this paper has the following interesting features. (1) Its particle content is chosen from six copies of the **27** and three

copies of the **27*** of E_6, consistent with the possibility that it is of superstring origin. (2) It is unifiable at around 10^{16} GeV even though the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ gauge symmetry is realized at the TeV energy scale with $g_L = g_R$. (3) The problem of requiring two or more scalar bidoublets for realistic quark masses in the conventional left-right model is circumvented because the $SU(2)_R$ doublet is not $(u,d)_R$, but $(u,h)_R$ where h is heavy and has lepton number $L = 1$. This allows the model to be free of tree-level flavor-changing neutral currents at the electroweak energy scale. (4) The τ lepton gets a mass spontaneously as usual, but e and μ acquire radiative masses through gaugino exchange. (5) All three neutrinos have small seesaw masses through their couplings to neutral gauge singlets with large Majorana masses. Additive lepton number L is explicitly violated but multiplicative lepton number $(-1)^L$ is preserved. (6) The structure of this model is naturally maintained with a discrete $Z_4 \times Z_2$ symmetry which is spontaneously broken down to a generalization of R parity. (7) At the TeV energy scale, there will be many unique manifestations of this model. For example, the W_R vector gauge boson here has a nonzero lepton number and negative R parity, hence its final decay product must contain at least a lepton as well as the LSP (lightest supersymmetric particle).

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FIGURE CAPTION

Fig. 1. Evolution of α_i^{-1} with $\mathrm{M_R} = 1$ TeV and $\mathrm{M_U} = 1.1 \times 10^{16}$ GeV.

Superfield	Z_4	Z_2
Q	1	$+$
Q^c	$-i$	$+$
d^c	1	$+$
h	-1	$+$
η	i	$+$
$\Phi_{L(1)}$	1	$+$
$\Phi_{L(2,3)}$	i	$-$
$\Phi_{L(4,5,6)}$	$-i$	$-$
$\Phi_{R(1)}$	$-i$	$+$
$\Phi_{R(2,3)}$	1	$-$
$\Phi_{R(4)}$	i	$-$
$\Phi_{R(5,6)}$	-1	$+$
$\Phi^c_{L(1,2,3)}$	i	$-$
$\Phi^c_{R(1)}$	$-i$	$-$
$\Phi^c_{R(2,3)}$	-1	$+$
$N_{L(1,2,3)}$	-1	$+$
$N_{R(1,2,3)}$	-1	$-$

Table 1: Transformation properties of the various superfields of this model under $Z_4 \times Z_2$.

Superfield	SU(3) × SU(2) × U(1)	B	L	R_f
(u, d)	$(3, 2, 1/6)$	$1/3$	0	$+$
d^c	$(\overline{3}, 1, 1/3)$	$-1/3$	0	$+$
u^c	$(\overline{3}, 1, -2/3)$	$-1/3$	0	$+$
h	$(3, 1, -1/3)$	$1/3$	1	$-$
h^c	$(\overline{3}, 1, 1/3)$	$-1/3$	-1	$-$
(η_1^0, η_1^-)	$(1, 2, -1/2)$	0	1	$+$
$(\phi^0_{L(2,3)}, \phi^-_{L(2,3)})$	$(1, 2, -1/2)$	0	1	$+$
ϕ_R^+	$(1, 1, 1)$	0	-1	$+$
$(\phi_R^c)^-$	$(1, 1, -1)$	0	1	$+$
N_L	$(1, 1, 0)$	0	-1	$+$
$\phi_R^0, (\phi_R^c)^0, N_R$	$(1, 1, 0)$	0	0	$-$
$\Phi_{L(1,4,5,6)}$	$(1, 2, -1/2)$	0	0	$-$
$(\eta_2^+, \eta_2^0), \Phi_L^c$	$(1, 2, 1/2)$	0	0	$-$

Table 2: Transformation properties of the various superfields of this model under SU(3) × SU(2) × U(1), B, L, and R_f.